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Calculating borehole geometry from standard measurements of borehole inclinometry

Kenneth C. Jezek and Richard B. Alley

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report describes two new methods for computing borehole geometry from discrete measurements of borehole inclination (θ) and azimuth (α). In the first method θ and α are assumed to vary linearly with arc length. This results in an analytic model of the borehole that is continuous but not smooth. The second model, which takes θ and α to vary quadratically with arc length between three measuring points, improves the smoothness of the model but the analysis must be carried out numerically. These models were applied to the		

20. Abstract (cont'd)

borehole inclinometry data collected at DYE-3, Greenland. The methods were found convenient to use and it is claimed that the results represent physically reasonable approximations to the borehole geometry.

PREFACE

This report was written by Dr. Kenneth C. Jezek, Geophysicist, Geophysical Sciences Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory, and Richard B. Alley, Research Assistant, Geophysical and Polar Research Center, Department of Geology and Geophysics, University of Wisconsin-Madison. This study was supported by National Science Foundation grant DPP 83-06689.

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CALCULATING BOREHOLE GEOMETRY FROM STANDARD MEASUREMENTS
OF BOREHOLE INCLINOMETRY

Kenneth C. Jezek and Richard B. Alley

INTRODUCTION

The most common method for estimating borehole geometry in glaciers is standard borehole inclinometry. Inclinometry data collected during the drilling operation are used to monitor the progress of the drill (Gundestrup et al. 1982) and eventually are used to orient and interpret the stratigraphy in the core. Subsequent remeasurements of borehole geometry are the most accurate methods for estimating intraglacial strain rates (Savage and Patterson 1963, Garfield and Ueda 1976).

Inclinometry logs measure local changes in the inclination and azimuth of the borehole. A method successfully used by Gundestrup and Hansen (in press) to log the deep-drill hole at DYE-3 on the Greenland Ice Sheet employs two pendulum type inclinometers with their sensitive axes orthogonal. A magnetic compass is used to determine the magnetic azimuth of the sensitive axes. Measurements are made at discrete points spaced on the order of tens of meters along the borehole and are converted to values of borehole inclination, azimuth and arc length. (Nearly continuous yet rapid logging is possible with a new technique that tracks a radar target lowered into the borehole [Jezek, in press], but the method is not yet as accurate as the standard procedure.) The precision and reproducibility of these measurements are very good -- 1% of the measured inclination and $\pm 2^\circ$ in the azimuth* -- but a dilemma arises in choosing an analytical method for computing the coordinates by integrating the inclination and azimuth data. Several methods can be found in the literature of the petroleum industry (Rivero 1971, Blythe 1975, Taylor and Mason 1971).

In this paper we will describe the details of a method we used in analyzing inclinometry data collected in the borehole at DYE-3, Greenland. Our model is similar to that proposed by Walstrom et al. (1972) (their model 4)

*Personal communication with B.L. Hansen, of the Polar Ice Coring Office, Lincoln, Nebraska, 1984.

and makes the simple assumption that inclination and azimuth vary linearly with arc length between measuring points. Our model suffers from a universal flaw in these approaches, which is the ad hoc assumption about how the borehole varies between measuring points. However, we claim that our method can be used to create physically reasonable models even in cases where the borehole is believed to behave in a complex fashion. Furthermore, we feel our technique is convenient and straightforward to understand.

METHOD

We assume that the inclination θ and azimuth α are different linear functions of arc length S between measurement points,

$$\theta = aS + \theta_0 \quad (1a)$$

$$\alpha = cS + \alpha_0 \quad (1b)$$

where

$$a = \frac{\theta_1 - \theta_0}{\Delta S} \quad (2a)$$

$$c = \frac{\alpha_1 - \alpha_0}{\Delta S} \quad (2b)$$

and where θ_0 and α_0 are the inclination and azimuth at the top of an interval of arc length ΔS , and θ_1 and α_1 are the inclination and azimuth at the bottom of that interval. We imagine the borehole as a smooth, three-dimensional curve and describe it by a set of three equations parameterized against arc length (Fig. 1),

$$dz = \cos \theta dS \quad (3a)$$

$$dx = \cos \alpha \sin \theta dS \quad (3b)$$

$$dy = \sin \alpha \sin \theta dS. \quad (3c)$$

Now it is straightforward to substitute eq 1 into eq 3 and integrate. The result is that the Cartesian coordinates x , y and z at the bottom of the interval of arc length are given by

$$\begin{aligned} x = x' - P(A + B) + Q(E + F) - R(E - F) - T(A - B) \\ + P \left(\frac{a}{(a-c)(a+c)} \right) + T \left(\frac{c}{(a-c)(a+c)} \right) \end{aligned} \quad (4a)$$

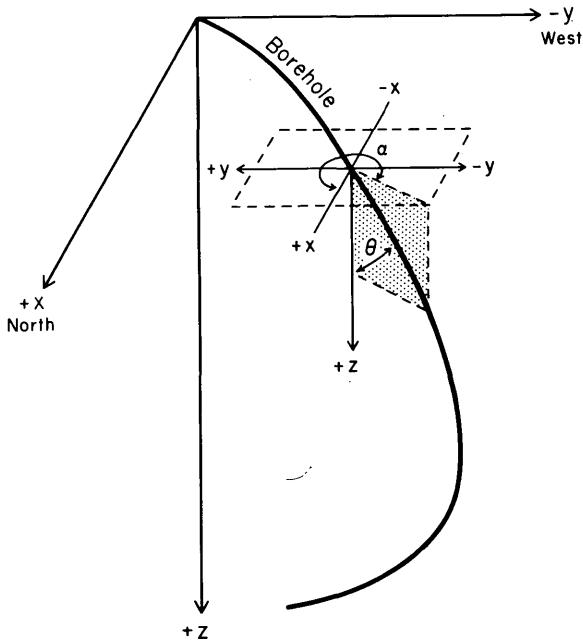


Figure 1. Geometry for computing the coordinate transformation. The x-y plane at $z = 0$ represents the surface of the glacier. The heavy, curved line represents the borehole. The azimuthal angle α is measured relative to the positive x direction (which is true north for the coordinates of the DYE-3 borehole data). The inclination angle θ is measured relative to the z-axis.

$$y = y' + P(E - F) + Q(A - B) - R(A + B) + T(E + F) \quad (4b)$$

$$- Q \left(\frac{c}{(a-c)(a+c)} \right) + R \left(\frac{a}{(a-c)(a+c)} \right)$$

$$z = z' + \frac{\sin(a\Delta S + \theta_0)}{a} - \frac{\sin \theta_0}{a} \quad (4c)$$

where

$$P = \cos \alpha_0 \cos \theta_0 \quad (5a)$$

$$Q = \cos \alpha_0 \sin \theta_0 \quad (5b)$$

$$R = \sin \alpha_0 \cos \theta_0 \quad (5c)$$

$$T = \sin \alpha_0 \sin \theta_0 \quad (5d)$$

and

$$A = \frac{\cos [(a-c) \Delta S]}{2(a-c)} \quad (6a)$$

$$B = \frac{\cos [(a+c) \Delta S]}{2(a+c)} \quad (6b)$$

$$E = \frac{\sin [(a-c) \Delta S]}{2(a-c)} \quad (6c)$$

$$F = \frac{\sin [(a+c) \Delta S]}{2(a+c)} \quad (6d)$$

and where x' , y' and z' are the Cartesian coordinates at the top of the interval. The equations for x and y are different for the case where $a = c$; then

$$x = x' + \frac{\sin^2(a\Delta S)}{2a} (P - T) + \frac{\Delta S}{2} (Q - R) + \frac{\sin(2a\Delta S)}{4a} (Q + R) \quad (7a)$$

$$y = y' + \frac{\Delta S}{2} (P + T) + \frac{\sin(2a\Delta S)}{4a} (T - P) + \frac{\sin^2(a\Delta S)}{2a} (Q + R). \quad (7b)$$

The equation for z is different for the case when $\theta_0 = \theta_1$ and $a = 0$; then

$$z = z' + S \cos \theta_0. \quad (8)$$

This model, like most other published models, assumes that all information about borehole orientation along an interval is contained in data measured at the endpoints of that interval. In some cases, borehole orientation may be related to the orientation in the previous interval. This is likely to be true for a new borehole in a glacier because drilling tends to progress along a smooth curve, but may be less likely for an old borehole where any shear deformations (which could be caused by material discontinuities such as dust) will distort the borehole in an unpredictable way. If the deformation rate is small, then a preferred model might be quadratic expressions for θ and α such as

$$\theta = eS^2 + fS + \theta_0 \quad (9a)$$

$$\alpha = gS^2 + hS + \alpha_0 \quad (9b)$$

where

$$e = \frac{\theta_1 - 2\theta_0 + \theta_{-1}}{2\Delta S^2} \quad (10a)$$

$$f = \frac{\theta_1 - \theta_{-1}}{2\Delta S} \quad (10b)$$

$$g = \frac{\alpha_1 - 2\alpha_0 + \alpha_{-1}}{2\Delta S^2} \quad (10c)$$

$$h = \frac{\alpha_1 - \alpha_{-1}}{2\Delta S} \quad (10d)$$

and where θ_{-1} and α_{-1} are the inclination and azimuth at the point above the interval in question and where ΔS is assumed constant for the length of the borehole. Equation 9 can be substituted into eq 3 and integrated numerically to trace the borehole. (We note that if borehole orientation in an interval is related to even longer lengths of borehole, then recursion formulae for higher order polynomials can be generated for computing the transformation.)

DISCUSSION AND CONCLUSIONS

We entered the equations describing linear variations of θ and α on a programmable hand calculator and used a simple difference test to choose whether eq 4, 7 or 8 would be used for each interval (Appendix A). As an example, we applied the method to the logging data (Gundestrup and Hansen, in press) from the DYE-3 borehole (Fig. 2). We averaged data where multiple measurements were available and used smooth curves to extrapolate θ and α from the shallowest measurements at the 100-m depth to the surface. The results are shown in Figure 3.

We also compared the linear-variation and quadratic-variation models for part of the borehole (Appendix B). The Cartesian coordinates generated by quadratic variations are only about 0.5% different from those generated by the linear-variation model (Table 1).

A drawback to the linear-variation model, which is minimized somewhat in the quadratic model, is that the variation of θ and α along the entire length of the borehole is continuous but not smooth. This, however, is a considerable improvement over other methods (such as assuming that the angles of inclination and azimuth measured at the bottom of the interval are valid over

Table 1. Comparison of linear-variation and three-point quadratic-variation models for 1900 to 1925-m depth. Run with $n=6$ in quadratic model.

s(m)	Inclination θ°	Azimuth α°	$\Delta y(m)$		Difference $\left(\frac{\text{Quad-Lin}}{\text{Quad}}\right) \times 100\%$
			Linear	Quadratic	
1895	8.84	282.8	-	-	-
1900	9.07	281.7	-	-	-
1905	8.83	283.6	-0.758	-0.762	0.53%
1910	9.01	280.6	-0.757	-0.753	-0.54%
1915	8.84	282.8	-0.759	-0.763	0.48%
1920	8.94	282.1	-0.754	-0.751	-0.34%
1925	8.785	281.0	-0.754	-0.756	0.22%
			Σ -3.785	-3.788	0.07%

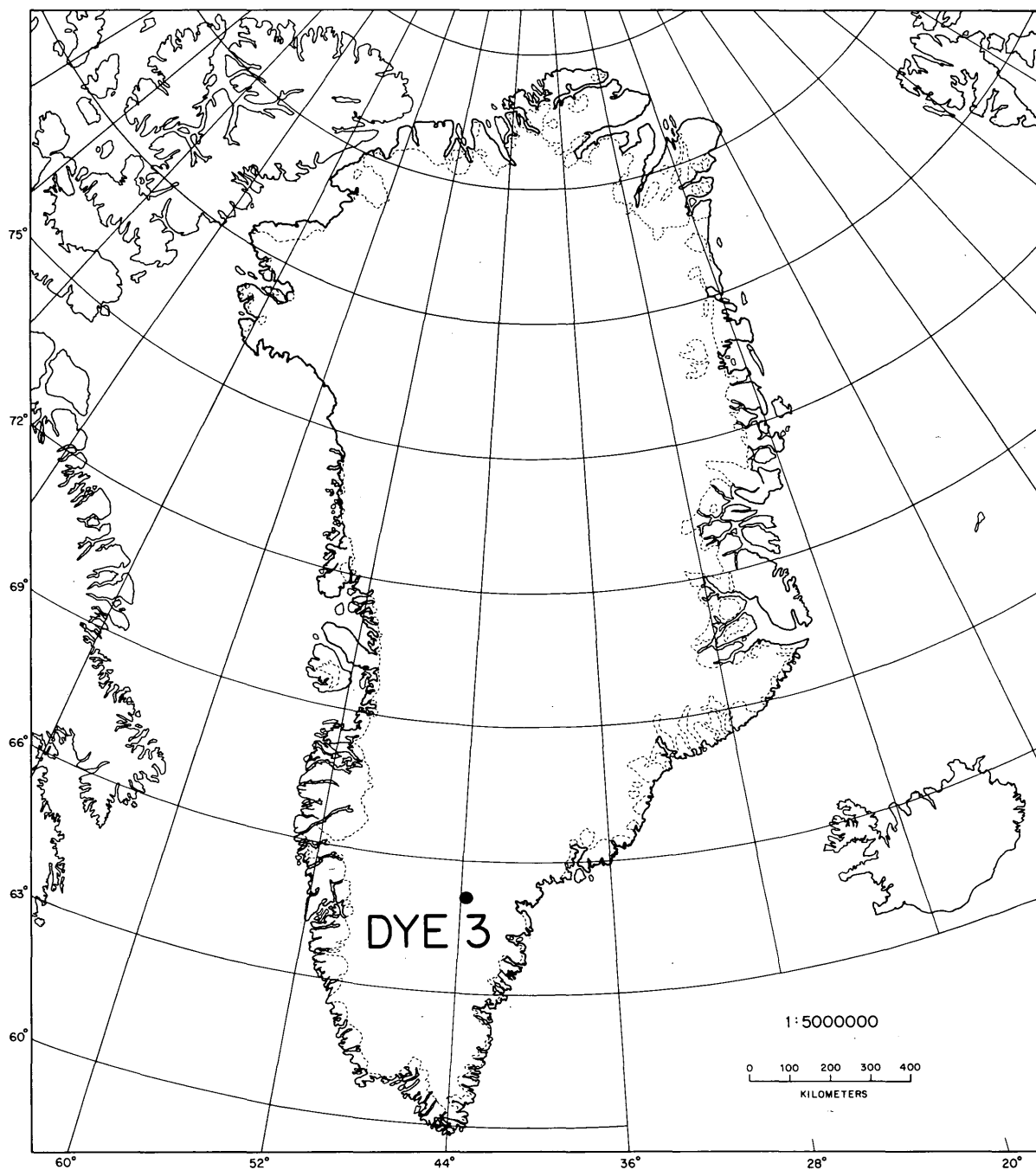


Figure 2. Location of DYE-3 in Greenland.

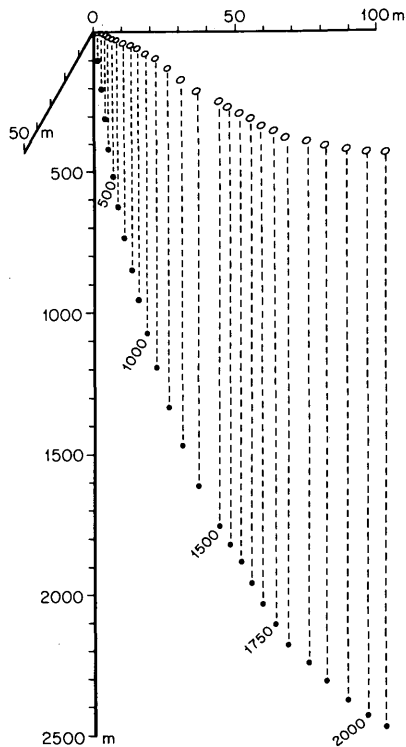


Figure 3. Spatial coordinates of the DYE-3 borehole computed by applying the linear-variation model to the inclinometry data collected by Gundestrup and Hansen (in press). The open ellipses represent the projection of the borehole onto the surface of the glacier. The solid circles represent points along the borehole at different depths.

the entire interval or using average values for the up-segment and down-segment coordinates based on the angles measured at the end points of the segment) which produce models of θ and α that are not continuous.

Our purpose has been to describe the transformation of azimuth, inclination and arc-length data to Cartesian coordinates, rather than to examine a particular logging experiment. Thus, we limit our conclusions to the statement that the linear-variation method is conceptually simple, straightforward to implement, and that it can be expanded to nonlinear variations.

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APPENDIX A: TEXAS INSTRUMENTS TI-59 CALCULATOR
PROGRAM FOR LINEAR-VARIATION MODEL

Memories are numbered 01 to 39.

Keystrokes preceded by the shift (2nd) key are underlined.

Abbreviations used:

- INV - inverse (of the following keystroke).
- $x \div t$ - puts displayed number into test register.
- $x > t$ - transfers control to the following location if the displayed value (x) is greater than or equal to the value in the test register (t); otherwise, control does not pass to the indicated location but goes to the next statement.
- INV $x > t$ E x^2 - will pass control to statements following label E if x is less than t; otherwise, it will square x.
- cos - cosine.
- sin - sine.
- STO xx - stores the displayed value in memory xx.
- SUM xx - adds the displayed value to the value in memory xx, and stores the sum in memory xx.
- RCL xx - recalls the value in memory xx.
- GTO label - GO TO label, unconditionally transfers control to the indicated location.
- LBL - Label.
- OP xx - special calculator operation (such as partitioning the calculator between addressable memories and program steps).
- R/S - run/stop; stops operation if calculator is running.
- *
- 4 Op 17 - partitions calculator for 39 memories, 639 steps.
- Rad - puts calculator into radians mode.

Enter program into calculator.

Store θ_0 (in radians = degrees * $\pi \div 180$) in memory 02.

Store α_0 (in radians = degrees * $\pi \div 180$) in memory 04.

Store θ_1 (in degrees) in memory 26.

Store α_1 (in degrees) in memory 27.

Store Δs (in meters) in memory 05.

Store 0.0001 in memory 30 - used in test to decide which sets of equations to use.

Press A.

The calculator displays Δx (in meters).

The calculator contains Δx , Δy and Δz in memories 20, 21 and 22 respectively.

For subsequent intervals - enter new θ_1 and α_1 into memories 26 and 27 (in degrees):

- Enter new Δs (this is only necessary if Δs changes)

- Press A.

After each calculation, the calculator displays Δx for that interval:

- Memories 20, 21 and 22 contain Δx , Δy and Δz , respectively, for that interval.

- Memories 23, 24 and 25 contain x , y and z ($\Sigma \Delta x$, $\Sigma \Delta y$ and $\Sigma \Delta z$), respectively, to the bottom of that interval.

Keystroke
number for
first stroke
on line

Keystrokes

Purpose

000	<u>LBL</u> A	
002	RCL 30 x \div t	put value from 30 into test register
005	RCL 02 STO 01	old θ_1 becomes new θ_0
009	RCL 04 STO 03	old α_1 becomes new α_0
013	RCL 26 * $\pi \div 180 =$ STO 02	calculate θ_1 in radians
024	RCL 27 * $\pi \div 180 =$ STO 04	calculate α_1 in radians
035	(RCL 02 - RCL 01) \div RCL 05 = STO 06	store a in 06
048	(RCL 04 - RCL 03) \div RCL 05 = STO 07	store c in 07
061	RCL 06 - RCL 07 = STO 08	store (a-c) in 08
069	RCL 06 + RCL 07 = STO 09	store (a+c) in 09
077	0.5 \div RCL 08 = STO 10	store 1/2(a-c)
085	0.5 \div RCL 09 = STO 11	store 1/2(a+c)
093	(RCL 08 * RCL 05) <u>cos</u> * RCL 10 = STO 12	store A in 12
107	(RCL 09 * RCL 05) <u>cos</u> * RCL 11 = STO 13	store B in 13
121	(RCL 08 * RCL 05) <u>sin</u> * RCL 10 = STO 14	store E in 14
135	(RCL 09 * RCL 05) <u>sin</u> * RCL 11 = STO 15	store F in 15
149	RCL 03 <u>cos</u> * RCL 01 <u>cos</u> = STO 16	store P in 16
159	RCL 03 <u>cos</u> * RCL 01 <u>sin</u> = STO 17	store Q in 17
169	RCL 03 <u>sin</u> * RCL 01 <u>cos</u> = STO 18	store R in 18
179	RCL 03 <u>sin</u> * RCL-01 <u>sin</u> = STO 19	store T in 19
189	RCL 08 x ² \sqrt{x}	calculate $ (a-c) $

Keystroke number for first stroke on line	Keystrokes	Purpose
193	INV $x \gt t$ E	if (a-c) is not zero, calculate Δx and Δy using the following equations. If (a-c) equals zero (is within 0.0001 of zero) calculate Δx and Δy using the equations at label E
196	RCL 16 * (RCL 10 + RCL 11 - RCL 12 - RCL 13) + RCL 17 * (RCL 14 + RCL 15) - RCL 18 * (RCL 14 - RCL 15) - RCL 19 * (RCL 12 - RCL 13 - RCL 10 + RCL 11) = STO 20	calculate Δx and store in 20
254	RCL 16 * (RCL 14 - RCL 15) + RCL 17 * (RCL 12 - RCL 13 - RCL 10 + RCL 11) - RCL 18 * (RCL 12 + RCL 13 - RCL 10 - RCL 11) + RCL 19 * (RCL 14 + RCL 15) = STO 21	calculate Δy and store in 21
312	<u>LBL</u> B	
314	RCL 06 $x^2 \sqrt{x}$	calculate $ a $
318	INV $x \gt t$ D	if a is within 0.0001 of zero, Δz is calculated using the equation at D; otherwise, Δz is calculated using the following equation
321	((RCL 06 * RCL 05 + RCL 01) <u>sin</u> - RCL 01 <u>sin</u>) \div RCL 06 = STO 22	calculate Δz and store in 22
344	<u>LBL</u> C	
346	RCL 20 SUM 23	calculate $x (= \Sigma \Delta x)$ and store in 23
346	RCL 20 SUM 23	calculate $x (= \Sigma \Delta x)$ and store in 23
350	RCL 21 SUM 24	calculate $y (= \Sigma \Delta y)$ and store in 24
354	RCL 22 SUM 25	calculate $z (= \Sigma \Delta z)$ and store in 25
358	RCL 20	display Δx
360	R/S	stop

Keystroke
number for
first stroke
on line

Keystrokes

Purpose

361	<u>LBL</u> E	for $ a-c \approx 0$
363	(RCL 06 * RCL 05) <u>sin</u> x ² ÷ 2 ÷ RCL 06 = STO 31	store $\frac{\sin^2(a\Delta s)}{2a}$ in 31
380	(2 * RCL 06 * RCL 05) <u>sin</u> ÷ 4 ÷ RCL 06 = STO 32	store $\frac{\sin(2a\Delta s)}{4a}$ in 32
398	RCL 05 ÷ 2 = STO 33	store $\frac{\Delta s}{2}$ in 33
405	RCL 16 - RCL 19 = STO 34	store (P-T) in 34
413	RCL 17 + RCL 18 = STO 35	store (Q+R) in 35
421	RCL 34 * RCL 31 + RCL 35 * RCL 32 + RCL 33 * (RCL 17 - RCL 18) = STO 20	store Δx in 20
446	RCL 31 * RCL 35 - RCL 32 * RCL 34 + RCL 33 * (RCL 16 + RCL 19) = STO 21	store Δy in 21
471	GTO B	return control to location B
473	<u>LBL</u> D	routine calculates Δz for $ a \approx 0$
475	RCL 01 <u>cos</u> * RCL 05 = STO 22	Δz in 22
484	GTO C	return control to location C

APPENDIX B: TEXAS INSTRUMENTS TI-59 CALCULATOR PROGRAM FOR QUADRATIC-VARIATION MODEL, ALLOWING Δs TO VARY.

Notation is as before, plus

PGM xx - program xx in Master Library Solid State Software module.

INV SBR - inverse subroutine returns control from a program (or other subroutine) to the calling routine.

This program uses the Master Library PGM 09, Simpson's Approximation (continuous) to evaluate the integral that arises. Because of restrictions in the logic of PGM 09, it is necessary to calculate Δx (or Δy or Δz , but only one) down the entire borehole, and then change the argument of the integral to calculate Δy or Δz . Running time for the program varies with the number of intervals (n) chosen for Simpson's approximation. Typical values for n and running time t are given below (for calculation of Δy between 1900 and 1905 m depth at DYE-3).

n	t(s)
2	21
4	27
6	36
10	50
20	88

We find n=6 to be a good compromise between speed and accuracy. Increasing n from 6 to 20 gave a change of only $1.56 \times 10^{-6}\%$ in Δy at 1900-1905 m. In many cases n=4 is quite accurate (to within 0.01% or better).

Model used:

Integrate from 0 to Δs_2 (from y_0 to y_1):

$y = \phi, \alpha$

$$y = \left[\frac{\Delta s_1 (y_1 - y_0) - \Delta s_2 (y_0 - y_{-1})}{\Delta s_1 \Delta s_2 (\Delta s_1 + \Delta s_2)} \right] s^2 + \left[\frac{\Delta s_1^2 (y_1 - y_0) + \Delta s_2^2 (y_0 - y_{-1})}{\Delta s_1 \Delta s_2 (\Delta s_1 + \Delta s_2)} \right] s + y_0$$

if $\Delta s_1 = \Delta s_2$

$$y = \left[\frac{y_1 - 2 y_0 + y_{-1}}{2 \Delta s^2} \right] s^2 + \left[\frac{y_1 - y_{-1}}{2 \Delta s} \right] s + y_0$$

Set calculator in radians (press rad).

Enter program into calculator.

Store θ_{-1}, θ_0 (in radians) in memories 11 and 12.

Store θ_1 (in degrees) in 13.

Store α_{-1}, α_0 (in radians) in 14 and 15.

Store α_1 (in degrees) in 16.

Store Δs_1 and Δs_2 in 17 and 18, respectively.

Store 0 in 01 (lower limit of integration for calculating Δx , Δy or Δz). Store n (we prefer $n=6$) in 28.

Press A.

After about 30 seconds, calculator displays Δx (for Δy or Δz , depending on the argument programmed for the integral).

Memory 29 contains the running sum of Δx (or $\Sigma \Delta y$ or $\Sigma \Delta z$).

Program automatically reassigns $\theta_0 \rightarrow \theta_{-1}$, $\theta_1 \rightarrow \alpha_0 \rightarrow \alpha_{-1}$, $\alpha_1 \rightarrow \alpha_0$; so, to calculate next step, simply store θ_1, α_1 (in degrees) for new step in memories 13 and 16, respectively, and press A. Note that if $\Delta s_1, \Delta s_2$ change, it is necessary to store the correct new values in memories 17 and 18.

Note that this procedure is designed to work down (or up) the hole and will not apply to the first interval at the top (or bottom) of the hole. The value of Δx (or Δy or Δz) can be calculated using the linear-variation routine and can be stored in memory 29, so that memory 29 contains the true running value of the coordinate being calculated. Alternately, one can reverse direction (if calculating downhole, calculate the first segment going uphole) and store the negative of the resulting Δ value in memory 29. Thereafter, one follows the instructions at the top of this page.

Keystroke
number for
first stroke
on line

Keystrokes

Purpose

000	<u>LBL</u> A	
002	RCL 13 * $\pi \div 180 =$ STO 13	Change θ_1 from degrees to radians
013	RCL 16 * $\pi \div 180 =$ STO 16	Change α_1 from degrees to radians
024	RCL 17 * RCL 18 * (RCL 17 + RCL 18) = STO 19	
040	(RCL 17 * (RCL 13 - RCL 12) - RCL 18 * (RCL 12 - RCL 11)) \div RCL 19 = STO 20	
069	(RCL 17 * (RCL 16 - RCL 15) - RCL 18 * (RCL 15 - RCL 14)) \div RCL 19 = STO 22	
098	(RCL 17 x^2 * (RCL 13 - RCL 12) + RCL 18 x^2 * (RCL 12 - RCL 11)) \div RCL 19 = STO 21	
129	(RCL 17 x^2 * (RCL 16 - RCL 15) + RCL 18 x^2 * (RCL 15 - RCL 14)) \div RCL 19 = STO 23	
160	RCL 28 STO 05	store n in 05 for discrete integration
164	RCL 18 STO 02	store Δs_2 in 02 for discrete integration (integrate from 0 to Δs_2)
168	RCL 18 \div RCL 05 - STO 03	calculate h (discrete interval width) and store in 03
176	<u>PGM</u> 09 D	call integration program and integrate
179	RCL 04 SUM 29	update running sum in 29 of Δ values
183	RCL 12 STO 11	reassign $\theta_0 \rightarrow \theta_{-1}$
187	RCL 13 STO 12	reassign $\theta_1 \rightarrow \theta_0$
191	RCL 15 STO 14	reassign $\alpha_0 \rightarrow \alpha_{-1}$
195	RCL 16 STO 15	reassign $\alpha_1 \rightarrow \alpha_0$
199	RCL 04	display calculated Δ value
201	R/S	stop

Keystroke
number for
first stroke
on line

Keystrokes

Purpose

To calculate Δx

202 LBL A'

204 ((STO 10 x^2 * RCL 22 + RCL 10 *
RCL 23 + RCL 15) cos * (RCL 10 x^2 *
RCL 20 + RCL 10 * RCL 21 +
RCL 12) sin) calculate Δx

243 INV SBR return control to main program

To calculate Δy (identical to Δx except cos \rightarrow sin)

202 LBL A'

204 ((STO 10 x^2 * RCL 22 + RCL 10 *
RCL 23 + RCL 15) sin * (RCL 10 x^2
* RCL 20 + RCL 10 * RCL 21 +
RCL 12) sin) calculate Δy

243 INV SBR

To calculate Δz

202 LBL A'

204 ((STO 10 x^2 * RCL 20 + RCL 10 *
RCL 21 + RCL 12) cos) calculate Δz

224 INV SBR