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September 1974

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HANOVER, NEW HAMPSHIRE

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.
ENGLISH TITLE: THERMAL DRILLING OF THE GLACIER

FOREIGN TITLE: ТЕРМОЧЕЛОДОЧЕРЕДНОЕ БУРЕНИЕ ЛЕДНИКОВЫХ ПОЛОВН ПОЛОВ

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SOURCE: Antarktika, Doklady Komissii, No 11, Moscow, Izdatel'stvo "Nauka", 1972, pp 141-156


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The basic knowledge concerning the structure of the deep layers of the ice sheet in the Antarctic and the processes occurring there has been obtained from direct measurements in boreholes and studies of core samples. However, until the present time, the main obstacle to such studies has been the difficulty in drilling adequately deep boreholes in the ice.

Thermal drilling is one of the techniques for drilling the ice, acquiring ever greater popularity in the Antarctic. The idea of obtaining boreholes in the layer of glaciers by thermal drilling has attracted the attention of researchers for a long time owing to the natural tendency to utilize the surprisingly low melting temperature of ice as a rock, forming the glaciers. The first successful experiments in thermal drilling of glaciers were conducted by Blumke and Hess in the Alps as early as the beginning of this century (Paterson, 1969).

Later on, a number of researchers utilized thermal drilling for obtaining boreholes both in the ice of mountain glaciers as well as in Greenland and the Antarctic. However, notwithstanding the relatively long history of this question, until the present time the technique of thermal drilling of glaciers has been in a state of experimental verification of various ideas and types of drills. Until now, there has been no standard portable thermal drill for drilling glaciers to relatively slight (to hundreds of meters) depths nor any stationary drill for the reliable obtainment of deep (up to thousands of meters) boreholes in the ice.
The reason for such a slow progress in the development of equipment for the thermal drilling of glaciers is found on the one hand, in the fact that the overly simple general idea of thermal drilling is built on the simplicity of its technical realization. Therefore, the thermal drills are frequently made by primitive methods by nonspecialists and the operations with them were discontinued after the initial tests demonstrated that the technical embodiment of the simple idea requires significantly more attention and time than was assumed at the beginning of the work. On the other hand, relatively little attention was paid to a theoretical analysis of the conditions involved in the operation of a thermal drill in ice, specifically to a study of the temperature perturbations in the ice, produced during thermal drilling, and so forth. In connection with this, among some specialists there existed the danger that the thermal drill would produce an excessively great disruption of the temperature regime in the glacier near the borehole. This risk also delayed the work in the area of utilizing the thermal drills.

The purpose of the present report is to attempt to classify the basic methods involved in the thermal drilling of glaciers, to present certain findings in the operations on the thermal drilling of glaciers in the Antarctic, and also to analyze theoretically the basic processes governing the thermal drilling conditions during passage through ice and typifying the conditions in the ice layer both ahead of the moving thermal drill as well as along the side walls of the borehole.

TECHNIQUES FOR THE THERMAL DRILLING OF ICE

We can differentiate three types of the thermal drilling of ice.

1. Drilling with the aid of a jet of gas or high velocity liquid without melting the ice. Under conditions when the temperature of gas or liquid differs from that of the ice, the high heat exchange coefficients caused by the high rates of motion of gas or liquid along the ice surface lead to the appearance of such considerable temperature gradients at the surface of the face that the ice can disrupt owing to the thermal stresses caused by these gradients and can be transported from the borehole without having reached the melting point. Moreover, the temperature of the jet can be either higher or lower than the temperature of ice at the surface of the face. This technique of thermal drilling is now gaining
considerable popularity in cutting boreholes in rocks (Brichkin, 1961; Brichkin, Belenko, 1961; Yagunov and others, 1962; Yagunov, 1963); however, the method has not yet been utilized for cutting through the ice. The relatively slight difference in the ice temperature under natural conditions from the melting point constitutes one of the obstacles in the application of the method indicated for drilling ice since it limits the range of temperatures within the limits of which the gradient occurs, determining the value for the disruptive thermal stresses.

2. Drilling of ice with the aid of a jet of gas or liquid owing to the melting or evaporation of ice at the face surface. Moreover, the temperature of gas or liquid should exceed the temperature of melting or evaporation of ice (of water). The case under review is also typified by extremely high coefficients of heat yield to the ice surface; however, the gradients developing thereby are insufficiently high to disrupt the ice at the surface of the face up to its melting or evaporation. In recent times, the drilling of such a type on a glacier has been accomplished with the aid of a jet of high temperature combustion products obtained in an air-gasoline burner. It was specifically by such a technique that S.V. Mikheev obtained boreholes in ice with a depth to 53 m (Mikheev, 1971). Operations are underway also in using charges of solid rocket fuel for the same purposes.

3. Drilling by melting of ice in the face owing to contact with the hard surface of the heater. At this time, the transmission of heat from the heater surface to the melting ice surface in the face is achieved with direct contact of the hard surfaces of the heater and ice, or by the transfer of heat through the layer of water having formed from the melting of ice between the heater and the face's surface. Thereby, the temperature of the heater surface is maintained fairly high by the continuous supply of energy to the heater. This can be electrical or chemical power or the energy from radioactive decay. This procedure is utilized most frequently for the thermal drilling of ice in the Antarctic. Usually the heating head of the drill is lowered into the borehole on a rope or cable, the reserve of which is kept at the surface. At this time, the borehole should be maintained for the period necessary for conducting the scientific observations. Such a drilling is achieved most simply through the layers of snow and firn usually forming the upper part of the layer of the Antarctic icecap. The drill used for such an operation is usually quite simple and is comprised of a fairly heavy frame with electric heating units installed in it. The energy necessary for maintaining the required temperature of the
heater frame is fed via a cable from the ice surface. It was specifically with such a technique that in 1959, V.S. Ignatov (1960) drilled a borehole at the Vostok Station; others who utilized such a drill included Yu.A. Kruchinin at Lazarev Station, A.P. Kapitsa and A.V. Krasnushkin at Drigal'skiy Island, and in 1960 - L.A. Dubrov in at Lazarev Station (Kruchinin, 1965).

Such thermal drills cut a glacier fairly reliably only to a depth of about 40 m. At these depths, the layer of the glacier is formed of snow and firn. Moreover, the water forming from the melting of ice runs completely through the pores into the layer of the glacier in the direction away from the face, leaving the borehole dry and cementing its walls. At greater depths in connection with the change in the structure of the layer, this water no longer runs through the pores and in proportion to cutting deeper, the heater proves to be submerged in the water having formed from the melting. The cutting rate decreases sharply; difficulties develop in the extraction of the heater since the water freezes and the heater turns out to be frozen into the ice by its upper end. In this way, the potentialities of the simplest thermal drills for cutting into cold and Arctic ice are extremely limited.

The necessity for obtaining deeper boreholes penetrating through the cold ice compels us to complicate the design of thermal drills with a system for pumping the melt water from the borehole. In the thermal drills of this type, we usually envisage the possibility of extracting a core. These thermal drills permit us to accomplish the drilling of boreholes in the Antarctic icecap for its entire depth (Barkov, 1960; Equipment for the Drilling of Boreholes ..., 1961). The initial tests on the investigation and development of a design for such drills with attachments for pumping out water and extracting a core in the Soviet Antarctic Expedition were started by N.I. Barkov in 1960 and were continued by V.A. Morev in 1961 (Morev, 1961; Barkov, 1963). In 1966, Ye.V. Kudryavtsev and A.V. Sekurov (Kudryavtsev, Sekurov, 1966; Sekurov, 1967) conducted near the Mirny Station tests of a drill with an attachment for the pumping of water and the extraction of the core, constructed for the Soviet Antarctic Expedition in the Moscow Institute of Radioelectronics and Mining Electromechanics. In addition, three boreholes were cut with a total length of 101 m. The maximal depth of boreholes was 77 m (the drill reached the rock bottom). In 1968, using an improved design of a similar drill, V.A. Morev at a distance of 25 km from the Mirny Station cut a borehole to a depth of 212 m. In this same year, B.B. Kudryashov, V.F.
Fisenkov and other researchers constructed in the Leningrad Mining Institute imeni Plekhanov under assignment by the AASRI (Arctic and Antarctic Scientific Research Institute) a new thermal drill with an attachment for pumping water and extracting a core sample. In 1965, V.F. Fisenkov drilled a borehole 50 km from the Mirnyy Station to a depth of 250 m. In 1970, using a drill of this type, N.I. Barkov began cutting a deep borehole in the central part of the Antarctic at the Vostok Station. Toward the end of 1970, the depth of this borehole exceeded 500 m.

In the U.S., such a drill was built in the Cold Regions Laboratory and tested in Greenland, where in 1965 it cut a borehole of 537 m (Grosval'd, 1969; Ueda, Donald, 1968).

In recent times, special interest has been developed in the thermal drill, which is lowered into the ice only under the effect of its own weight, not leaving boreholes behind it. In this case, the entire supply of cable is accommodated in the upper part of the actual drill and is unwound as it lowers. At this time, a borehole is not maintained since the water occurring in it re-freezes. The drill is reminiscent of a spider which is descending its web. It is natural that such a drill should carry an entire set of instruments required for taking measurements in the ice layer during the passage and after stoppage of the drill. Such a drill was tested successfully in Greenland and in 1968 it cut a borehole to a depth of 1000 m (Aamot, 1970).

**THERMAL REGIME OF OPERATION OF A THERMAL DRILL; EFFICIENCY**

Let us consider the uniform movement of a thermal drill with a solid heater in solid ice. In the ice region in which the effect of the heater is exerted, we will differentiate two zones. Zone I is located directly under the heater and comprises a cylinder with a diameter equaling that of the heater. Zone II is the zone of ice situated to one side of Zone I (Fig. 1).

Under the effect of the heater's weight, its lower heated surface transmits heat via water gap A to the ice surface of face B. This heat is used in warming the ice in Zone I from its natural temperature to melting point $q_{m}$, on the actual melting of ice in Zone I at a rate equaling the heating rate $q_{rx}$ on the heating of water in the gap between the heater...
and face $q_{x}$. Part of the heat ($q_{x}$) is spent in melting the side walls of the borehole; this leads to an increase in the diameter of the borehole as compared with that of the heater. The heat is also spent in heating the side walls of borehole ($q_{r}$). Heat $q_{r}$ and $q_{x}$ is spent in Zone II. Let us signify by $P$ the thermal force of the heater. Then at established motion of the heater into the ice:

$$P = q_{x} + q_{r} + q_{b} + q_{LR} + q_{IR}. \tag{1}$$

Here the value $q_{x} + q_{r}$ = the useful expenditure of heat, i.e. the heat which is necessary for drilling. The ratio of useful expenditure of heat to the entire amount of heat released by the heater is the efficiency ($\eta$) of the thermal drill. In this way, the thermal drill's efficiency is determined by the following ratio:

$$\eta = \frac{q_{x} + q_{r}}{P}. \tag{2}$$

In its nature, this expression corresponds to the one for the efficiency of the heater in an electrothermal drill (Kudryavtsev, Sekurov, 1966). For an actual determination of the efficiency of a thermal drill and the other parameters characterizing the drilling process, we require a detailed analysis of the values of each of the terms in Eqs. (1) and (2). Such an analysis has been made below.
TEMPERATURE IN ICE AHEAD OF A MOVING THERMAL DRILL

The first effort at evaluating the temperature fields in ice during thermal drilling was made by S.S. Sillin (1965). However, this study was devoted chiefly to an analysis of the "acceleration" process not established in time; under the conditions of our problem, this process is negligibly minor.

It is feasible to investigate the motion of a thermal drill in ice under the conditions of a temporarily-established process in a mobile system of coordinates shifting in the ice along with the drill. For an analysis of the heat transfer in Zone I ahead of the moving drill, let us consider the one-dimensional problem concerning the advancing of a melting front in an unlimited medium at a constant rate equaling $u$.

Let us direct the $x$-axis along the heater's axis in the direction of the front's movement. Let us assume for the origin of coordinates the displacing melting front. Then the temperature field ahead of the melting front will be described by the equation

$$a \frac{d^2 t}{d x^2} + u \frac{dt}{dx} = 0$$

at the boundary conditions $x = 0, t = t_{in}, x = \infty, t - t_0$; here $a$ = coefficient of ice's temperature conductivity; $t$ = temperature in point at distance $x$ from the melting front; $t_{in}$ = temperature of ice's melting; $t_0$ = temperature of ice not disturbed by the influence of thermal drill.

The solution to this equation determines unequivocally the temperature field in the glacier in front of the melting surface and has the form:

$$\frac{t - t_{in}}{t_0 - t_{in}} = 1 - \exp \left( - \frac{u}{a} x \right).$$

In Fig. 2, we have shown the temperature distribution curve ahead of the melting front for various rates of the thermal drill's movement into the ice, having an initial temperature of $-50^\circ$. The melting point of ice is assumed to equal $0^\circ$. As is evident in the graph, at travel rates of around 0.1 cm/sec, close to the cutting rates of modern thermal drills, temperature at a distance of around 1 cm from the front differs very little from the natural, original ice temperature.
This conclusion testifies to the slight disruption of the thermal condition of ice by the thermal drill; this is important in estimations of the possibility of employing the drill for glaciological investigations.

![Graph](image)

Fig. 2. Temperature Distribution in Ice Layer Ahead of Thermal Drill at Various Cutting Rates.  
1 - \( u = 0.01 \text{ cm/sec} \); 2 - \( u = 0.03 \text{ cm/sec} \); 3 - \( u = 0.06 \text{ cm/sec} \); and 4 - \( u = 0.1 \text{ cm/sec} \).

The flow of heat diverted into the ice from the melting front \((q_{x})\) with consideration of Eq. (4) equals:

\[
q_{x} = -\pi R_{o}^2 \frac{d}{dx} \left[ (t_{0} - t_{in}) \left( 1 - \exp \left( -\frac{u}{c} \right) x \right) \right] = ct_{u} \left( t_{in} - t_{b} \right) \pi R_{o}^2,
\]

(5)

where \( \pi R_{o}^2 \) = area of thermal drill mid-section; \( t_{u} \), \( c \) = heat capacity and density. As could have been expected, the flow of heat diverted from the melting front into the ice owing to heat conductance, under the conditions of the one-dimensional problem, is expended entirely in raising the heat content of ice, arriving at the melting front, to the temperature of phase transition. For a drill with a central opening for the core, Eq. (5) should be multiplied by the value \( l = \frac{R_{o}^2 \beta_{H}}{R_{o}^2} \), where \( R_{o} \) = dimensionless radius of opening for taking the core.

TEMPERATURE OF ICE AT WALLS OF BOREHOLE OBTAINED BY THERMAL DRILLING TECHNIQUE

The temperature in the ice in the direction away from the face is higher in Zone II than the temperature of the undisturbed ice. The amount of heat \( q_{x} \) is spent in heating
the side walls of the borehole. For determining the temperature field in these walls and the amount of heat diverted into Zone II in a direction away from Zone I bounded by the lateral surface of borehole \( R = R_0 \), let us revert to the problem concerning the advance of a point source of heat placed along the axis of a heater and shifting downward at constant speed \( u \).

We will consider that the strength of this source is maintained in the plane \( a - a \) (refer to Fig. 1) at distance \( R_0 \) from the heater axis, a temperature equaling that of ice melting. The problem concerning the distribution of temperatures in the layer around a point source moving in a body at a constant speed has been solved by D. Rosenthal (1946). With consideration of the solution obtained by him and the remark made above, the temperature field in plane \( a - a \) perpendicular to the direction of the thermal drills motion can be represented as:

\[
\frac{t - t_0}{t_n - t_0} = \exp \left( -\frac{uR_0}{2a} \left( \frac{R}{R_0} - 1 \right) \right),
\]

(6)

where \( \frac{R}{R_0} = \text{dimensionless distance from the borehole wall}. \) In Fig. 3, we have shown the curves reflecting the temperature variation as a function of distance from the borehole wall for various travel rates of the thermal drill (probe) into the ice. The curves are drawn for ice having an initial temperature of \(-50^\circ\) and a melting point of \(0^\circ\). The diameter \( 2R_0 \) of borehole is assumed to equal 100 mm. As is evident in the set of curves, the heating of the borehole wall as a result of drilling is also reflected only in a relatively shallow layer. Let us note that the influence of the borehole diameter within the limits of variation in \( R_0 \) from 2.5 to 10 cm is very slight and amounts to around 1\(^\circ\) at distances from the wall equaling 5 mm.

![Fig. 3. Temperature Distribution in the Ice Layer at Borehole Walls at Various Cutting Rates. 1 - \( u = 0.01 \) cm/sec; 2 - \( u = 0.03 \) cm/sec; 3 - \( u = 0.06 \) cm/sec; and 4 - \( u = 0.1 \) cm/sec. Key: a. cm.](image-url)
Fig. 4. Coefficient of Heat Dissipation in Direction from Face and its Dependence on Rate of Cutting and Radius of Thermal Drill. 1 - $R_o = 1 \text{ cm}$; 2 - $R_o = 2 \text{ cm}$; 3 - $R_o = 4 \text{ cm}$; 4 - $R_o = 5 \text{ cm}$; and 5 - $R_o = 8 \text{ cm}$. Key: $a, U, \text{ cm/sec}$.

The amount of heat having arrived into the ice in Zone II can be determined proceeding from the fact that it is expended entirely on changing the heat content of ice which passes at velocity $u$ through the plane $a - a$ and at this time alters its temperature from $t_o$ to $t$:

$$q_{xR} = \int_{R_o}^{\infty} \epsilon \gamma u (t - t_o) 2\pi R dR.$$  

Substituting into this expression the $t$-value and conducting a transformation, we obtain:

$$q_{xR} = (t_{in} - t_o) \epsilon \gamma u R_o \frac{\alpha t}{u R_o}.$$  \hspace{1cm} (7)

A comparison of the expression obtained with Eq. (5) determining the dissipation of heat into the ice in Zone I for heating the ice subjected to further melting indicates that Eq. (7) differs from Eq. (5) only by virtue of the dimensionless factor $K_R$:

$$K_R = \frac{\alpha t}{u R_o} \frac{1}{(t + R_{in})}.$$  \hspace{1cm} (8)

This factor determines the share of heat dissipated into the ice in Zone II in a direction from Zone I and can be referred to as the coefficient of heat dissipation into the walls. In Fig. 4, we have reflected the curves for the dependence of $K_R$ on the rate of the drill's movement and its diameter for $R_{in} = 0$ (drill without core). As is obvious in the graph, at low travel speeds of thermal drill and small
diameters of the heater, the effect of diverting the heat into
the side walls of the borehole becomes quite significant.

TEMPERATURE OF FRONTAL PART OF THERMAL DRILL

In spite of the fact that the temperature at the surface
of the ice face equals the melting point, the temperature in
the thermal drill wall and the temperature of water in the
gap between the heater and the face can differ appreciably
from the melting point of ice. A knowledge of this temperature
is necessary for explaining the safe operating conditions of
the heater and also for determining the amount of heat spent
in heating the water in the gap above the melting point $T_{m}$.

The temperature of heater surface is found from the
equation of heat balance at the surface ice melting under the
heater:

$$- \lambda_b \frac{dt}{dx} \bigg|_{x=H} = uT \left[ r + c(t_{na} - t_0) \right],$$

(9)

where $\lambda_b$ = the coefficient of water heat conductivity;
$\frac{dt}{dx} \bigg|_{x=H}$ = the temperature gradient in the water at the
ice surface (here and below, we consider a heater with a
forward heated surface which is plane and perpendicular to
the borehole's axis); and $r =$ latent heat of ice melting.

When the distribution of temperature in the depth of
the water layer in the gap between the heater and ice is
linear, condition (9) easily determines the temperature of
the heater's surface ($t_H$)

$$t_H = \frac{uT}{\lambda_b} \left[ r + c(t_{na} - t_0) \right] + t_{na},$$

(10)

where $H$ = thickness of water layer in the gap. However, the
presence of a vertical rate of water motion in the gap between
the heater and ice caused by the outflowing of water from the
melting surface does not permit us to consider the temperature
distribution through the depth of water layer as linear, even
under the assumption of constancy for the heat conductivity
factor in the water layer.

Let us consider the transport of heat from the heater
toward the ice through a water gap in the presence of the
indicated movement of water in the gap. We will consider that
the temperature of the forward surface of the heater is uniform.
in all its points while the thickness of the water layer between the heater and ice is constant. Let us consider an elementary column of water located along the heater axis. The origin of coordinates will be placed on the surface of the heater; let us as formerly direct the x-axis downward. In the adopted system of coordinates, the travel rate of the liquid in direction x will vary from zero at the heater surface to a value equaling velocity \( w \) of water outflow from the surface of ice thawing in direction x, i.e. to a value differing from the cutting rate \( u \) by the value of ratio of densities of water \( \gamma_d \) and of ice \( \gamma_i \); \( w = u \gamma_i / \gamma_d \). For simplification of the problem, let us assume the distribution of velocities of water travel in the gap in the direction x as linear and satisfying only the conditions at the boundaries: \( x = 0, 2w^* = 0, x = H; \) and \( 2w^* = u \gamma_i / \gamma_d \). In this case, the transport of heat in direction x is established by the following equation:

\[
\frac{dt}{dx} + \frac{1}{\gamma_n} \cdot \frac{dx}{a_H} \cdot \frac{dt}{dx} = 0
\]

(11)

with the boundary condition: \( x = 0, t = t_{in}; x = H, t = t_{out} \).

The solution to this equation describing the distribution of temperature gradients in the water layer has the form:

\[
\frac{dt}{dx} = \frac{t_{in} - t_{out}}{H} - \frac{2}{V \pi} \cdot \exp \left( \frac{\sqrt{\frac{uH}{2n_i \gamma_i}} \cdot x}{H} \right) \cdot \left[ \exp \left( \frac{uH}{2n_i \gamma_i} \cdot \frac{x}{H} \right) \right] \]

(12)

where \( erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp -f^2 df = \) the integral of errors (Yanke, Emde, 1959). Having assumed the value of temperature gradient of water at the boundary with thawing ice \( dt/dx \bigg|_{x=H} \) from

Eq. (12) after transformation we obtain the following expression for determining the temperature of heater surface, with consideration for the motion of water in the gap

\[
t_n = \frac{uH}{2n_i \gamma_i} \cdot \left[ \frac{V \pi \cdot \exp \left( \frac{uH}{2n_i \gamma_i} \cdot \frac{x}{H} \right)}{2} \right] \cdot \left[ \exp \left( \frac{uH}{2n_i \gamma_i} \cdot \frac{x}{H} \right) \right] \cdot \left[ t_{in} - \frac{t_{out}}{H} \cdot \left( \sqrt{\frac{uH}{2n_i \gamma_i}} \right) \right] \]

(13)

Let us point out that the factor enclosed in brackets in the right-hand part of Eq. (13) is greater than unity. From this we obtain the conclusion that the surface temperature of a heater moving at speed \( u \) into the ice at temperature \( t_0 \) is higher than follows from Eq. (10). Eq. (10) overlooks the effect of the dissipation of water from the surface of melting.
ice, since it was obtained under the assumption that the entire flow of heat diverted from the heater surface arrives at the surface of thawing ice.

As can be observed from Eq. (12), the temperature gradient in the water layer at the heater surface (at $x = 0$) is higher than the temperature gradient at the ice surface ($x = H$) and hence, the flow of heat diverted from the heater surface is greater than the heat flow expended for the actual melting of ice. At this time, some of the heat is spent for heating the water forming during the melting of ice. In this manner, the transfer of heat from the heater to the melting surface is achieved with a certain efficiency $\eta_1$ less than unity. Its value equaling the ratio of the amount of heat arriving at the ice, to the amount of heat diverted from the heating surface of the thermodrill equals:

$$\eta_1 = \frac{q_{\text{avg}} - q_{\text{avg}}}{\frac{aH}{x_0}} \cdot \exp \left( -\frac{aH}{2a_0} \cdot \frac{x}{x_0} \right).$$ \hspace{1cm} (14)$$

As is evident from Eqs. (13) and (14), the temperature of the heater surface and the efficiency of transmitting the heat via the water layer depend appreciably on the depth of this layer.

![Graph](image)

**Fig. 5.** Relationship of Surface Temperature of Leading Wall of Heater on Drilling Speed, Depth of Water Layer and Cutting Rate. Key: a. $uH$, cm²/sec.

We have indicated in Fig. 5 the dependence of the temperature of the heaters' surface and efficiency $\eta_1$ of the drill's head section upon the value of products $uH$. As is evident from the graph, at low values for $uH$, we can disregard the heating of the water layer. This permits us to determine the surface temperature of the heater based on a simple equation (10).
DEPTH OF WATER LAYER BETWEEN HEAD SECTION OF THERMAL DRILL AND ICE

If as a first approximation we consider as negligibly slight the effect of frictional forces in the water layer between the drill nose and ice, the depth of water layer will then be determined from the conditions of equality of working of gravitational force of the heater descending in the gravitational field and the kinetic force of water which is flowing from under the heater. For simplicity of the discussion, let us assume that in the center of the thermal drill there is no hole for extracting a core for the sampling of water which has formed. We can then consider that the water forming from the melting of ice during cutting escapes from under the heater only along the borehole walls. During the motion of such a drill at velocity \( u \) and its weight \( G \), this equality of energies can be represented in the form

\[
u G = \frac{r_2}{g} \cdot \frac{W_{R_o}^2}{2} \cdot \frac{R_o}{2\pi R_o H W_{R_o}}.
\]

where \( g = \) acceleration of gravitational force; \( H = \) depth of water layer under the heater; \( W_{R_o} = \) average (through the depth of water layer) rate of its radial flow for a distance \( R_o \) from the center, i.e., in the place where the water escapes from the space between the head part of the heater and the thawing ice into the region between the side walls of the heater and the borehole walls (it is assumed that the clearance between the walls of the borehole and the side surface of the heater is sufficiently great). The value for velocity \( W_{R_o} \) in Eq. (15) is determined from the equation of continuity which in integral form is expressed as:

\[
\pi u R_o^2 \cdot W_{R_o} 2\pi R_o H = \pi u R_o^2 \cdot W_{R_o} 2\pi R_o H.
\]

Substituting this expression into Eq. (15), after transformations, we obtain the following formula for determining the depth of water layer between the head of heater and the ice:

\[
H = \sqrt{\frac{\pi}{u} \cdot \frac{W_{R_o}^2}{2\pi R_o}} - \frac{u}{g} \cdot \frac{r_2}{g}.
\]

A more precise estimation of the water layer depth should be conducted with allowance for the effect of water friction against the wall of ice and the heater.

The depth of water layer between the heater head and the ice, at steady conditions of heater's movement, is defined as the depth at which the force of pressure of the heater on
the ice is balanced by the pressure in the water layer, being forced out from under the heater after the water is formed. To generalize the discussion, let us assume that in the center of the thermodrill there is a hole for extracting a core or pumping water, such opening having the radius \( R_H \).

If \( p(R) \) is the pressure in the water layer in the gap, depending on distance \( R \) from the heater axis, while \( G \) is the pressure of the heater on the ice, the condition at which the depth of water layer does not change through time can be written:

\[
G = 2\pi \int_{R_H}^{R} p(R) R dR.
\]

(17)

The distribution of pressure in the water layer depends significantly on the layer's depth and is determined in the general formulation from the motion equations of energy and continuity in the water layer. This problem has been reviewed in detail by R. Shreve (1962), but in the general formulation of the question which Shreve utilized, he failed to obtain simple mathematical expressions for finding the link between the depth of water layer with other parameters determining the drilling. Therefore the results of his study have been presented in the form of tables obtained by calculation on electronic computers. However, it appears feasible to us to again examine the problem, having in mind the introduction of such limitations or related assumptions which would permit us to obtain mathematical formulas establishing the relationship of the water layer depth with the parameters typifying the thermal drills.

Following R. Shreve, we note that the variation in pressure in the water layer in direction \( x \) can be disregarded as compared with the variation in pressure in radial directions. In the given formulation, in distinction from that previously reviewed by us, we will disregard the inertial terms. Then for the axial-symmetric problem, the equation of motion in cylindrical coordinates has the form:

\[
H^2 \frac{\partial p(R)}{\partial R} = \frac{\partial}{\partial x} \left[ \mu(t) \frac{\partial v}{\partial x} \right].
\]

(18)

Here \( \bar{x} = x/H \) = dimensionless distance from the heater surface; \( \mu(t) \) = viscosity of water depending on temperature and, therefore, variable over the depth of water layer; and \( v \) = velocity of radial spreading of water in the gap.
The conditions on the surfaces of heater and ice are the conditions of adhesion: $x = 0, v = 0; x = 1, v = 0.$

In Fig. 6, we have shown the curve for the relationship of the coefficient of dynamic viscosity on temperature based on data furnished by Bingham and Johnson (Kay, Libby, 1962). As is evident from the curve, the variation in water viscosity with temperature is quite significant; therefore, it is necessary to solve Eq. (18) in combination with the heat transfer equation (11). R. Shreve proceeded in just this way. However, our evaluations indicated that in the problem under review, the distribution of viscosity through the depth of water layer can be assumed close to a linear one

$$\mu(x) = \mu_0 + (\mu_r - \mu_0) x,$$  \hspace{1cm} (19)

where $\mu_0 =$ water viscosity at its freezing temperature; $\mu_r =$ water viscosity at surface temperature of heater. The assumption indicated the necessity of involving Eq. (11).

A solution to Eq. (18), where viscosity depends only on the coordinate according to Eq. (19), was obtained by us in the form:

$$v = \frac{\partial p(R)}{\partial R} \frac{1}{H^2} \left[ \frac{x \ln \frac{\mu_r}{\mu_0} - \ln \left( 1 + \frac{\mu_r - \mu_0}{\mu_r} x \right)}{\mu_r \ln \frac{\mu_0}{\mu_0}} \right].$$  \hspace{1cm} (20)

At slight drops in temperature in the layer when the effect of temperature on water viscosity can be disregarded ($\mu_r \rightarrow \mu_0$), Eq. (20), applying the L'Hôpital rule twice, can be simplified to the Poiseuille equation

$$v = \frac{\partial p(R)}{\partial R} \frac{1}{H^2} \frac{1}{3\mu_0} (\hat{x}^2 - \hat{x}).$$  \hspace{1cm} (21)

The continuity equation for the given approximation can feasibly be presented in the following form:

$$2\pi RH \int_0^1 v d\hat{x} = \pi (R^2 - R_m^2) u \frac{\gamma}{\hat{r}_n}.$$  \hspace{1cm} (22)

Here, as in the case of R. Shreve, $R_m =$ radius on which there occurs the separation of melt water into the flow moving toward the outer edge of the drill head and the flow moving toward the central opening of the drill, if such is present.
Fig. 6. Water Viscosity as a Function of Temperature. On the y-axis — $\mu \cdot 10^2$ dynes $\cdot$ sec/cm$^2$

Fig. 7. Dependence of Coefficient $K_t$ on Temperature of Heater's Surface.

The pressure distribution along direction $R$ in the water layer can be determined as a result of a combined review of the equations for motion and continuity. In this context, let us assume the pressure in the water layer at the points of transition from the thermodrill head to its outer or inner lateral walls to be equal to atmospheric pressure. Let us tentatively assume that it equals zero. In this manner:

$$ R \cdot R_{in}, p \neq 0; \quad R \cdot R_{out}, p = 0. \quad (23) $$

Substituting Eq. (20) into expression (22) and integrating in respect to $x$ and $R$ with allowance for Eqs. (23), we derive an expression describing the distribution of water pressure (excess over atmospheric) in the layer between the melting ice and the forward flat surface of the heater:

$$ p = \frac{m_{in} T_n \left( \frac{R_o^2 - R_{in}^2}{2} - R_{in}^2 \ln \frac{R}{R_{in}} \right)}{2H^2 f \left[ \ln \frac{R_o}{R_{in}} - \frac{\mu_{in}}{\mu_{in} - \mu_{Rn}} \left( 1 + \frac{\rho_o - \rho_{in}}{\rho_{in}} \right) - \frac{\rho_o - \rho_{in}}{\rho_{in}} \ln \left( 1 + \frac{\rho_o - \rho_{in}}{\rho_{in}} \right) + 1 \right]} \quad (24) $$
where

$$R_m = \sqrt{\frac{R_o - R_{cm}}{2 \ln \frac{R_o}{R_{cm}}}}.$$  

By integration of Eq. (17) in which \( p \) is found with Eq. (24) we obtained the following relationship establishing the depth of water layer between heater and ice, with consideration of variable viscosity:

$$H = \left( \sqrt[3]{\frac{\mu_t \gamma T_{cm}^3}{8G \gamma}} \right) \cdot K_t \cdot K_o,$$  \hspace{1cm} (25)

where \( K_t = \) dimensionless coefficient taking into account the effect of heater's temperature on depth of water layer:

$$K_t = \sqrt[3]{\frac{1 - \frac{\mu_{nn}}{\mu_o} \ln \frac{\mu_{nn}}{\mu_{tn}}}{12 \ln \frac{\mu_{nn}}{\mu_{tn}} - \frac{\mu_{tn}}{\mu_o} - \frac{\mu_{nn}}{\mu_{tn}} (1 + \frac{\mu_{nn} - \mu_{tn}}{\mu_{tn}}) \ln (1 + \frac{\mu_{nn} - \mu_{tn}}{\mu_{tn}}) + 1}}.$$  \hspace{1cm} (26)

\( K_o = \) coefficient determining the effect of central opening on depth of water layer under the heater:

$$K_o = \sqrt[3]{\frac{4 + R_{nn} - 2 \frac{R_{nn}}{R_{cm}}}{\ln \frac{1}{R_{cm}}}}.$$  \hspace{1cm} (27)

Here \( \bar{R}_H = R_{nn}/R_o = \) dimensionless radius of central hole in frontal surface of the heater.

In Fig. 7, we have shown the curve reflecting the dependence of coefficient \( K_t \) on temperature at the heater surface obtained on the basis of Eq. (26) and with utilization of the relationship (shown in Fig. 6) of water viscosity to temperature. In Fig. 8, we have depicted the relationship curve for coefficient \( K_o \) determining the effect of the central hole on the depth of water layer. For practical purposes, it is advantageous to begin the calculation of the depth of water layer under the heater, assuming that water temperature in the gap does not vary along the depth of layer, and equals zero. Then the \( K_t \)-value equals unity. After this, based on Eq. (13), we determine the temperature of heater surface and the depth of water layer is directed by introducing coefficient \( K_t \) with the aid of the curve in Fig. 7. Then, again based on Eq. (10),
we determine the temperature of heater surface corresponding to the new value for the depth of water layer. Evaluations indicate that usually two repetitions of the indicated operations are adequate in order to obtain depths of water layer and temperature of heater surface close to the actual values.

THAWING OR FREEZING OF LATERAL BOREHOLE WALLS

A considerable share of the heater energy can be expended in the melting of the borehole's lateral walls. Under the conditions of a heater flooded by water, the transmission of heat for thawing the borehole walls is achieved not only by molecular heat conductance. In connection with the fact that the lateral heater walls have a temperature close to that of water boiling, we can expect the movement of liquid upward along the heater walls and its sinking along the cold borehole walls, and consequently, the additional heat exchange (associated with such movement) owing to the natural free convection. The heat flow through the layer of liquid between such walls can be computed utilizing the methods of stationary heat exchange through a medium in which heat transport by free convection is lacking. In this case, in place of the coefficient of molecular heat conductance $\lambda_{h}$, it is necessary to utilize the coefficient of equivalent heat conductance $\lambda_{B}$, taking into account the effect of free convection (Eckert, Drake, 1961).

For determining the depth of ice layer having thawed at the side surfaces of the heater for distance $h$ from the face, let us revert once again to Fig. 1. We will consider that the transport of heat in direction $R$ from the side wall of a heater having temperature $\tau_{r}$ toward the borehole wall with temperature $\tau_{w}$ is achieved through a water layer with effective heat conductance $\lambda_{B}$. In connection with the slight depth of layer, we can consider that the heat transfer in direction $x$ along the lateral surfaces is negligibly small vis-à-vis the transfer of heat athwart this layer. The slight depth of layer as compared with the thermodrill diameter also permits us to overlook the curvature of the side surfaces of the drill and borehole. Then the increase in the depth $\Delta l$ of water layer during the time $\Delta \tau$ on thawing of borehole's side surface owing to the transport of heat via a layer of depth is determined by the balance of heat flows passing through the layer and expended on thawing:

$$\lambda_{h} \cdot \frac{t_{w} - t_{na}}{t} \cdot \Delta \tau = \gamma_{s} \cdot (\Delta \tau) \cdot (t_{na} - t_{0}) \cdot \Delta l.$$
Fig. 8. Dependence of Coefficient $K_0$ on Relative Value for the Radius of Central Hole in Thermodrill $R_{H}/R_0$.

Fig. 9. Relationship of Value for Thawing of Side Walls of Boreholes ($\Delta R$) to the Value for the Heated Lateral Surface of Heater. 1 - according to Stefan; 2 - according to Eq. (34).

Decreasing the time $\Delta \tau$, passing to the limit and integrating, we obtain the following relationship linking the depth of water layer and the time during which the thawing took place:

$$\int_{0}^{1} \frac{1}{\lambda_{m}} \, dt = \frac{(t_n - t_{n,0}) \tau}{(t_n - t_{n,0}) \gamma_n}. \tag{28}$$

Let us note that here the value $\lambda_{m,0}$ is under the integral sign since in a general case it depends on many parameters, including the layer depth. The value $\lambda_{m,0}$ and its relationship with molecular heat conductance for various conditions have been derived by Krausold (Eckert, Drake, 1961), based on generalizing various experimental data concerning free convection in the layer of liquid between the plane or concentrically arranged vertical walls having varying temperature, in the form of the relationship:
\[
\frac{\lambda_n}{\lambda_{\text{n,0}}} = F(Gr, Pr),
\]

where \( Gr \) = dimensionless Grashof criterion determining the intensity of free convection and for our case having the form:

\[
Gr = \frac{g \beta 1(t_n - t_{\text{n,0}})}{\nu^3},
\]

where \( Pr = \text{dimensionless Prandtl criterion and } Pr = c_B \mu / \lambda_B \).
Here \( c_B = \text{heat capacity, } \mu \text{ and } \lambda_B = \text{the coefficient of dynamic viscosity and molecular heat conductance of liquid, respectively and } \beta = \text{coefficient of water's thermal coefficient.} \)

According to Krausold's data, the analysis conducted by us on Eq. (29) showed that under the conditions of a water-filled gap between the heater and wall of the borehole, with a temperature drop of around 50\(^o\)C, free convection is already manifested significantly at layer depth of around 0.5 mm. In this connection, for practical purposes within the limits \((t_H - t_{\text{r,0}}) \) from 50 to 100\(^o\)C, after finding the values for the dimensionless parameters, Eq. 29 can be estimated by a linear equation of the form

\[
\frac{\lambda_n}{\lambda_{\text{n,0}}} = 1 + a t.
\]

(30)

For water, the coefficient \( a \) is close to 6.5 cm\(^{-1}\).

Substituting Eq. (30) into Eq. (28) and integrating, we get the following expression determining the relationship of the depth \( l \) of ice layer having melted at the side wall of the borehole, to time \( \tau \) during which the borehole wall was under the influence of the heater (its wall temperature \( t_H \)):

\[
\tau = \left[ \frac{l}{2} - \frac{\ln(1 + 2t)}{2^2} \right] \frac{t [t + \epsilon (t_{\text{nw}} - t_0)]}{(t_{\text{nw}} - t_0) \lambda_n}.
\]

(31)

The effective time of the heater with temperature \( t_H \) as it influences the borehole side surface is determined by the travel rate of the heater into the ice, i.e., by the velocity \( u \) of cutting the borehole and by the distance of the borehole point under discussion to face h (see Fig. 1): \( \tau = h/u \). Substituting the last expression into Eq. (31) we obtain a dependence for finding the nature of increase in the thickness of lateral clearance between the heater and borehole:

\[
h \cdot \frac{(t_{\text{nw}} - t_0) \lambda_n}{u [t + \epsilon (t_{\text{nw}} - t_0)]} \left[ \frac{l}{2} - \frac{\ln(1 + 2t)}{2^2} \right]
\]

(32)
The second term in the left-hand part of Eq. (32) can be referred to as the coefficient of intensity \( \xi \) of walls' melting. Let us note that when the heat conductance of a liquid is constant and does not depend on depth of layer (there is no free natural convection), Eq. (28) leads to the familiar Stefan equation

\[
\tau = \frac{\mu}{2} \cdot \frac{\gamma (r + c(t_{nn} - t_0))}{(t_n - t_{nn}) \lambda_0} \tag{33}
\]

or

\[
h \cdot \frac{t_n - t_{nn}}{\mu \gamma (r + c(t_{nn} - t_0))} = \frac{\mu}{2} \tag{34}
\]

In Fig. 9, we have portrayed the relationship between the values of the gap formed on thawing of the borehole's side walls and distance \( h \) from the heater's lower end. The curve has been drawn on the basis of Eq. (32) for \( \alpha \) equaling 6.5. For the sake of comparison, in the same place we have drawn a curve of a similar relationship, constructed according to the Stefan equation (34). The X-axis on the graph shows the actual distances, i.e., the lengths of the thermodrill's surface below water level in the face (h) multiplied times the melting intensity factor \( \xi \). Such a scale provides a clear concept concerning the effect of various factors on the rate of the side walls' thawing. A knowledge of the depth of ice layer thawing from the side walls of the borehole is also adequate for determining the heat flow \( q_{HR} \) diverted from the heater for the thawing of side walls. In a first approximation, this flow is determined as:

\[
q_{HR} = 2\pi R \gamma (r + c(t_{nn} - t_0)).
\]

Speaking of the processes of heat exchange between the water and the borehole side, we must discuss the freezing of water at the side surfaces. A knowledge of the rate of such freezing is important in order to establish the length of thermodrill, with which its upper end will not freeze to the borehole walls. This is also important in designing a drill which is lowered into the ice without maintaining the borehole. The rate of water freezing in the space above such a drill determines the configuration and useful capacity of the container which can be moved along with the heater without expending additional energy for protecting the upper parts of the container (farthest from the heater) from freezing into the ice. The connection between the position of the limit of water freezing in the cavity, being determined by distance \( R_3 \) from
this boundary, to the axis of the drill and the time \( t_3 \) from
the start of freezing has been described approximately by the
expression developed by L.S. Leybenzon (1955):

\[
\frac{R_3^2}{2} \ln \left( \frac{R_3}{R_b} \right) = \frac{R_b^2 - R_3^2}{4} = \frac{\lambda_0 (t_0 + t_{\text{vec}})}{r_{\text{vec}}} r_b.
\]

CONCLUSIONS

In the report we have systematized the techniques
involved in thermal drilling of ice in glaciers and we have
conducted a theoretical analysis of the essential processes
determining the thermal drilling.

We have developed analytical expressions for establish-
ing the temperature field in ice ahead of the drilling front
and at the side walls of the borehole which is being drilled.
It has been demonstrated that a thermodrill produces very
slight disruptions to the temperature fields both ahead of
the drilling front and in the lateral borehole walls. These
disruptions decrease with an increase in the cutting rate and
at drill advancing speed of 0.1 cm/sec and more, the ice
temperature 5-7 mm ahead of the drilling front practically
equals the temperature of undisturbed ice.

We have developed relatively simple analytical equations
and graphs for computing the basic parameters determining the
thermal drilling: the relationships of drilling rate with
drill temperature depending on its configuration and weight,
dependences of the thawing of ice in the borehole side walls
on temperature of heater, its dimensions and cutting rate.

The data presented are useful both in the development
of new thermodrills and in the selection of the optimum con-
ditions of operating already existing equipment, and also in
analyzing the disturbances which are being caused during the
thermal drilling of ice.

BIBLIOGRAPHY

[1] Barkov, N.I., "Electric Drill for Cutting Boreholes in
Ice," Byull. Izobreteniya (Bulletin of Inventions), No 8,
1960.


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Unclassified

THERMAL DRILLING OF THE GLACIER

September 1974

Approved for public release; distribution unlimited.

Thermal drilling is one technique for drilling the ice in the Antarctic. The idea of obtaining boreholes in the layer of glaciers by thermal drilling has attracted the attention of researchers for a long time because of the tendency to utilize the low melting temperature of ice as a rock, forming the glaciers. This report attempts to classify the basic methods involved in thermal drilling of glaciers in the Antarctic, to present certain findings in the operations on the thermal drilling, and also to analyze theoretically the basic processes governing the thermal drilling conditions during passage through ice and typifying the conditions in the ice layer both ahead of the moving thermal drill as well as along the side walls of the borehole. Simple analytical equations and graphs for computing the basic parameters determining thermal drilling are presented. The data presented are useful both in the development of new thermodrills and in the selection of the optimum conditions of operating already existing equipment, and also in analyzing the disturbances which are being caused during the thermal drilling of ice.

14. KEY WORDS

DRILLING THERMAL DRILLING ICE DRILLS