ANTITORQUE LEAF SPRINGS

a design guide for ice-drill antitorque leaf springs

by

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Abstract: By shaping the leaf spring according to a forth order parabola, a uniform load distribution is obtained along the line of contact with the ice. For this leaf spring geometri, the radial load, the supporting force and the maximum bending moment in the spring are calculated.

Introduction

The leaf spring antitorque device is an improvement over most other designs, due to the following qualities:

a. Mechanically, it is very simple consisting of three pre-bent leaf springs, each supported at the ends by hinges.
b. The rise of the springs and thereby the pressure exerted on the hole wall, may easily be adjusted by changing the distance between the hinges.
c. The springs are flexible, allowing easy passage of e.g. ice layers or other irregularities at the wall of the drill hole.

The method of rational design of antitorque leaf springs presented here, was developed for the Danish light-weight drill (Johnsen, 1980). Experience with this drill and the Danish deep drill (Gundestrup, 1982) has lead to a satisfactory design procedure.

Leaf spring geometry

Figure 1 shows a sketch of the spring. In the figure,

r = radius of the hole,
b + e = radial distance of the spring supports (hinges) from the hole wall,
e = eccentricity,
2k = distance between the spring supports (assumed to be fixed).

The undeformed spring consists of three sections: two rectilinear ones adjacent to the supports of length c (projected length a, see figure 1), and a curved section with chord length 2l and rise f_l (thick full curve in fig.1).

The calculations are simplified, if the two points of transition between the spring sections (points C and D in figure 1) coincide with the hypothetical points of intersection between the undeflected spring and the hole wall. We assume this to be the case. The total rise of the spring will accordingly be $f_l + b$.

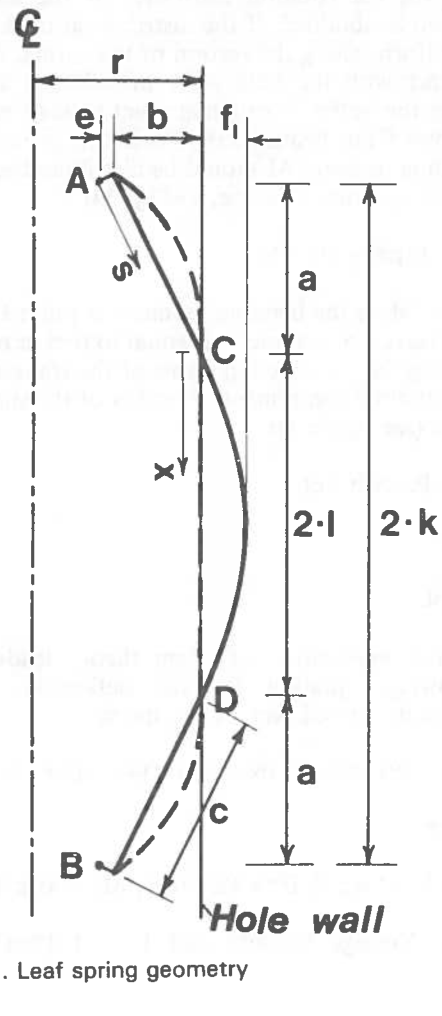

Figure 1. Leaf spring geometry

When the spring is deflected by being pressed against the hole wall, the curved section is straightened out and the rectilinear sections are bent (thick broken line in figure 1). The transition points C and D will be displaced a short distance along the hole wall in the direction towards the supports. Assuming the ratio $f_l/(2l) \ll 1$, the displacements are insignificant and are consequently ignored. This means, that the section of the deflected spring contacting the wall is assumed to have the length 2l, and that the chord lengths of the curved sections of the deflected spring are equal to c.

Design principles and equations

The following rules are used in the design of the leaf springs:

1. The load distribution should be uniform along the section of the deflected spring, that is in contact with the wall.
2. The transitions between the curved and rectilinear sections of the spring should be smooth, both in the deflected and undeflected states.
3. The length of the spring is preserved during deflection.
4. The deflections are calculated using ordinary beam theory.

The highest rotation resistance of the antitorque system is obtained, if the distribution of the load p is uniform along the section of the spring, that is in contact with the hole wall. Introducing an x-axis along the wall with origin at point C (see figure 1), it follows from beam theory, that the corresponding bending moment M should be distributed according to the equation (Hartog, 1949 p.34)

$$M = -\tfrac{1}{2}px^2 + plx + M_c, \qquad (1)$$

where M_c is the bending moment at point C (which for reasons of symmetry is equal to that at point D). M_c may be expressed in terms of the transverse and longitudinal components P and N of the supporting force (see figure 1):

$$M_c = Pa - N(b+e), \qquad (2)$$

where

$$P = pl. \qquad (3)$$

Further application of beam theory leads to the following equation for the deflections of the originally curved part of the spring

$$f/k = \gamma^3 P^*(1/24x^{*4} - 1/6x^{*3} + \tfrac{1}{3}x^*) + \gamma^2 M_c^*(x^* - \tfrac{1}{2}x^{*2}), \qquad (4)$$

where

$$\gamma = l/k,\ x^* = x/l,\ P^* = Pk^2/(EI),\ M_c^* = M_c k/(EI),$$

E = Youngs Modulu and I = $1/12wt^3$ is the moment of inertia of the spring cross section (w = width and t = thickness).

The expression for small deflections of a straight beam has been applied, even though the actual deflections are not small and the undeformed 'beam' is not straight. However application of a more correct - and correspondingly more complicated - theory is not likely to change the results significantly.

Eq. (4) ensures, that the deflections of points C and D are zero, in accordance with a previous assumption. Since section CD is deflected into a straight line, the shape of the curved central section of the undeflected spring is exactly given by eq. (4).

To ensure a smooth transition between the curved and the rectilinear sections of the undeflected spring, the slope of the curved spring section df/dx at points C and D should be equal to $\pm b/a$.

This imposes the following condition on P and M_c:

$$M_c^* = b^*/[\gamma(1-\gamma)] - \tfrac{1}{3}\gamma P^*, \qquad (5)$$

where $b^* = b/k$, and P^*, M_c^* and γ have been given above. Combining eqs. (2) and (5), yields

$$N^* = P^*(1-\tfrac{2}{3}\gamma)/(b^*+e^*) - b^*/[(b^*+e^*)\gamma(1-\gamma)], \qquad (6)$$

where

$$N^* = Nk^2/(EI) \text{ and } e^* = e/k.$$

Next consider section AC, along which an s-axis is introduced with origin at point A.

According to the theory of a beam subject to normal forces, the differential equation for the deflections of section AC will read (Hartog, 1949, pp.188-189)

$$EId^2f/ds^2 + f(Pb/c + Na/c) = Ne - (M_c + Ne)s/c,$$

which has the solution

$$f/k = f_s\sin(\omega s/c) + f_c\cos(\omega s/c) + a_0 + a_1 s/c, \qquad (7)$$

where

$$\omega^2 = [P^*b^* + N^*(1-\gamma)]\sqrt{(1-\gamma)^2 + b^{*2}}, \qquad (8)$$

$$a_0 = N^*e^*\{\omega/[P^*b^* + N^*(1-\gamma)]\}^2, \qquad (9)$$

$$a_1 = -(M_c^* + N^*e^*)\{\omega/[P^*b^* + N^*(1-\gamma)]\}^2, \qquad (10)$$

$$f_c = -a_0, \qquad (11)$$

$$f_s = -(a_1 + a_0)/\sin(\omega) + a_0\cos(\omega), \qquad (12)$$

Assuming k, b and e to be known, it appears that the deflection curve eq. (7) is dependent on the unknown quantities γ, P^*, N^* and M_c^* However, through eqs. (5) and (6), the number of unknowns may be reduced to two, e.g. γ and P^*. In order to determine these unknowns, two additional equations must be set up: One expressing, that the slope of the deflection curve given by eq. (7) equals $b^*/(1-\gamma)$ for s = c, the other expressing that the length of the spring is preserved. The two equations read:

$$\omega f_s\cos(\omega)-\omega f_c\sin(\omega)+a_1-b^*/(1-\gamma)\sqrt{(1-\gamma)^2+b^{*2}}=0, \quad (13)$$

and

$$\int_0^c\sqrt{1+(df/ds)^2}ds+1=\int_0^1\sqrt{(1+(df/dx)^2}dx+c, \quad (14)$$

Equations (13) and (14) are two transcendental equations in γ and P^*, that must be solved by numerical methods. The solutions are shown in figure 2 a and b for various values of the dimensionless parameters b/k and e/k.

In figure 2 are also plotted additional quantities, that are usefull in the leaf spring design. A practical design schema is given below.

The main characteristic lengths k, b and e of the leaf spring geometry are constrained by practical considerations. Next the dimensionless force N^* and the maximum bending moment M_s^* which occurres near the midpoint of section AC, are obtained from the diagrams shown in figure 2c) and d) and the maximum allowable thickness of the spring is calculated from the expression

$$\sigma_b/E=1/12(t/k)^2N^*+½(t/k)M_s^*, \quad (15)$$

where σ_b is the allowable bending stress of the spring material. In most practical cases t/k is of magnitude 1/100 and the N^*-term is negligible. In this case the maximum allowable thickness of the spring is given by the simpified expression

$$t=2k\sigma_b/(EM_s^*), \quad (16)$$

The value of P^* can now be found from the diagram shown in figure 2a, and the total radial force excerted by the antitorque system on the hole wall can be calculated as (assuming 3 springs):

$$P_r=½P^*Ewt^3/k^2, \quad (17)$$

If the approximate thickness given by eq. (16) is introduced, we get

$$P_r=4kw\sigma_b^3P^*/(E^2M_s^{*3}),$$

If this force - with a reasonable choice of the spring width w - is sufficient to prevent the drill from rotating, the next step will be to obtain γ from the diagram shown in figure 2b. Then l and a can be calculated, $l = \gamma k$ and $a = (1-\gamma)k$ respectively. Finally the shape of the curved section of the spring can be obtained by means of eq. (4), using the values of γ and P^* found above, and the value of M_c^* obtained from the diagramme shown in figure 2e. The rise of the curved spring section can be found from figure 2f.

If the radial force given by equation (17) is too small, different characteristic lengths k, b and e should be used, and the design procedure repeated until a satisfactory result is obtained.

The spring supports of course should be designed to carry the combined loads N and P.

Example

For the Danish ISTUK deep drill (Gundestrup, 1982) the characteristic lengths of the antitorque system are k = 34.5 cm, b = 3.7 cm and e = 0.6 cm, resulting in b/k = 0.107 and e/k = 0.0174.

With these values, the following dimensionless quantities are obtained from the diagrams shown in figure 2: $P^*=4.25$, $\gamma=l/k=0.452$, $N^*=20.4$, $M_s^*=-0.97$, $M_c^*=-0.208$, and $f_l/k=0.060$.

Taking w = 2 cm, t = 0.25 cm and $E=2.1*10^6$ kp/cm², the corresponding non-dimensionless quantities become: P = 19.5 kp, l = 15.6 cm, a = k-l = 18.9 cm, N = 93.7 kp, $\sigma=7580$ kp/cm² (eq.(15)), and $f_l=2.1$ cm.

Finally the shape of the curved section of the spring is found by means of eq.(4). The following table presents the results:

x/l	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
f(cm)	0	0.31	0.62	0.92	1.20	1.46	1.67	1.85	1.98	2.06	2.08

References

Gundestrup, N.S., S.J. Johnsen and N. Reeh, ISTUK, a deep ice core drill system, this volume.

Den Hartog,J.P., Strength of Materials, Dover Publications, Inc, 1949

Johnsen, S.J., W. Dansgaard, N.S.Gundestrup, S.B.Hansen, J.O.Nielsen, and N.Reeh, A fast light-weight core drill,*J.Glac.*, Vol.25, No.91, 169-174,1980.

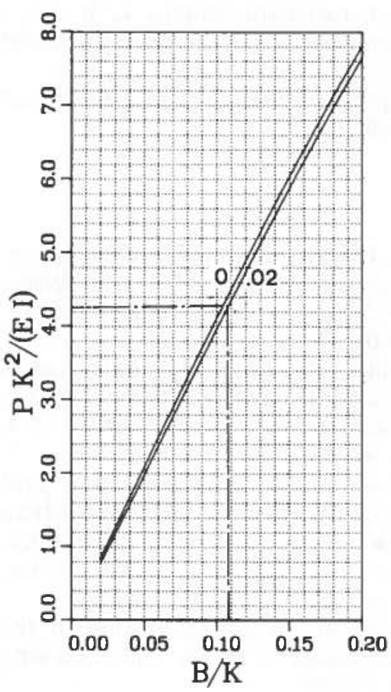

Figure 2a. Dimensionless transverse supporting force as function of b/k for the indicated values of the dimensionless eccentricity of the spring support.

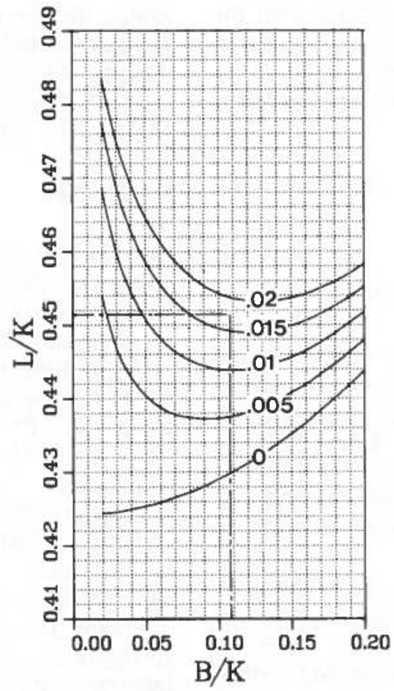

Figure 2b. Chord length of curved spring section as function of b/k for the indicated values of the dimensionless eccentricity of the spring support.

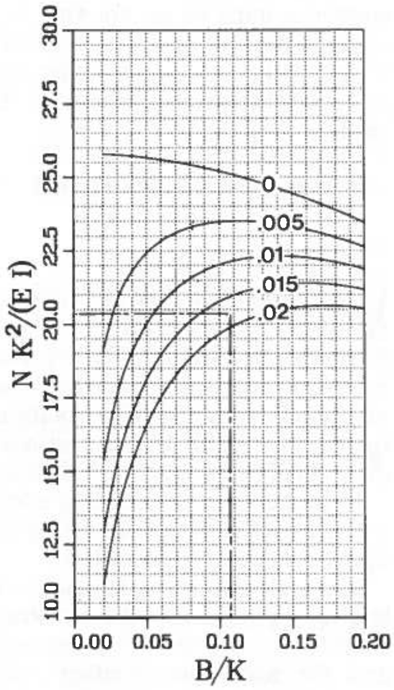

Figure 2c. Longitudinal supporting force as function of b/k for the indicated values of the dimensionless eccentricity of the spring support.

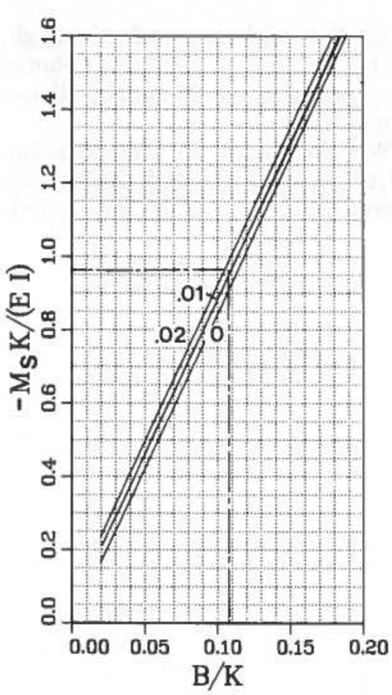

Figure 2d. Maximum absolute bending moment as function of b/k for the indicated values of the dimensionless eccentricity of the spring support.

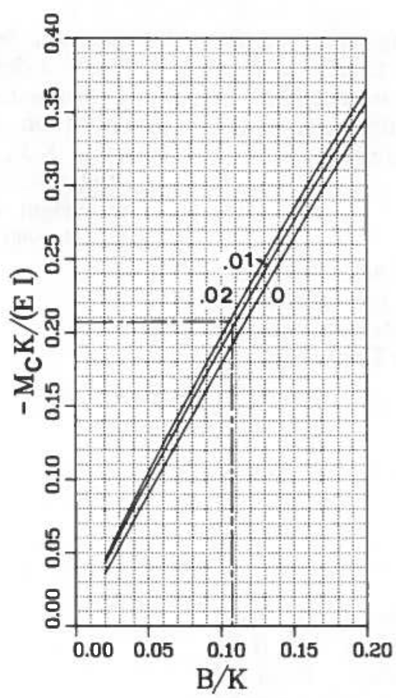

Figure 2e. Bending moment at transition point as function of b/k for the indicated values of the dimensionless eccentricity of the spring support.

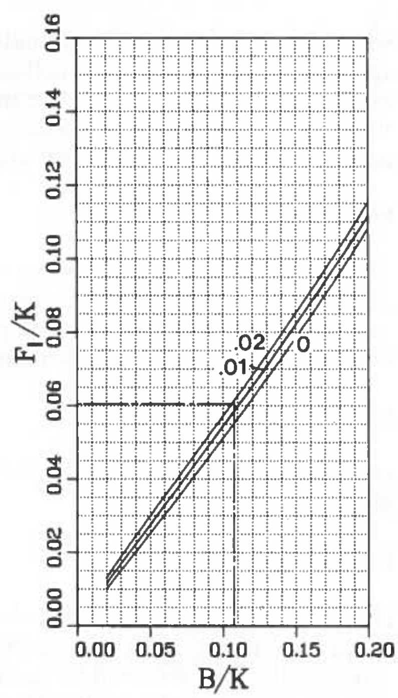

Figure 2f. Rise of curved spring section as function of b/k for the indicated values of the dimensionless eccentricity of the spring support.

Figure 2. Leaf spring design diagrams.